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Simple Sandwich Algebras

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Abstract—We describe simple sandwich algebras with maximal multiplication space.

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Natural and important algebraic systems possessing all the properties of rings, with the exception of the multiplication associativity, arose a very long time ago. In particular, the construction of the Lie algebra arose in connection with the problem of the solvability in quadratures of systems of differential equations. Later it was noted that the solutions to some systems of differential equations can be detected using a more general construction, which was called the Lie pencil [1, 2]. Lie pencils were first introduced in the work of I. L. Cantor, and D. B. Persitz [3].

We now give the definition of a Lie pencil. Let L be a finite-dimensional vector space over the field P. We denote by K the space of all bilinear skew-symmetric mappings from $L \times L$ to L.

Definition 1. A vector space L over a field P is called a Lie pencil, if K contains a subspace S such that for any $s \in S$ the relation

$$(asb)sc + (bsc)sa + (csa)sb = 0, \quad a, b, c \in L$$

holds. (Here xsy is the image of the pair $(x, y) \in L \times L$ under the mapping s.)

Such a sheaf we denote by $L(S)$, and S is called the multiplication space of this pencil.

Due to the results of [4, 5] the simple Lie pencils are often realized as sandwich algebras.

Definition 2. A sandwich algebra $M_n(L, S)$ is a pair of subspaces L, S from $M_n(P)$ such that $asb - bsa \in L$, whenever $a, b \in L$, $s \in S$.

(Here $M_n(P)$ is the space of matrices of order n over the field P, and asb and bsa are the standard associative products of three matrices.)

The sandwich algebra $M_n(L, S)$ can be regarded as a Lie pencil $L(S)$, assuming S to be a multiplication space of L.

In what follows we denote the commutator $asb - bsa$ by $[a, s, b]$.

The subspace I of the sandwich algebra $M_n(L, S)$ is called an ideal if $[a, s, b] \in I$ for any $a \in I$, $b \in L$, $s \in S$.

A sandwich algebra $C = M_n(L, S)$ is said to be simple if it does not contain nontrivial ideals and $C^2 \neq 0$. (The product of the spaces M and N in C is the linear span $\langle [a, s, b], a \in M, b \in N, s \in S \rangle_P$.)

Theorem 1. *Let I be the minimal left ideal of the algebra $M_n(P)$, and let L be an arbitrary nonzero subspace of I. Then the sandwich algebra $M_n(L, M_n)$ is simple.*

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